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ARISTIDES QUINTILIANUS AND CONSTRUCTIONS IN EARLY MUSIC THEORY

Aristides Quintilianus' dates are not known, but he can hardly be earlier than the first century A.D. or later than the third. Several passages in the early pages of his *de Musica*[1] purport to record facts about the practice of much older theorists, in contexts which make it clear that his references are to the period before Aristoxenus. Since our knowledge of music theory in that period is extremely sketchy, it is obviously worth trying to assess the reliability of Aristides' information. Two of his references have often been recognized as being of special interest, and there is a third, to which, I shall argue, the other two are intimately related. The first (12. 12 to the end of the diagram on 13) records two systems of notation, alleged by Aristides to have been used by οἱ ἀρχαῖοι. The second (18. 5 to the end of the diagram on 20) is the famous, or notorious, account of certain 'divisions of the tetrachord' which were employed by οἱ πάνυ παλαιότατοι πρὸς τὰς ἁρμονίας. It is these, Aristides tells us, which are mentioned by Plato in the *Republic*.[2] The remaining passage (15. 8–20) is superficially rather less exciting: it records the names and initial notes of the ἁρμονίαι, or forms of octave scale, said to have been distinguished by οἱ παλαιοί, and says something about a method by which the ποιότης of each can be made clear. The information given here about the nature of the ἁρμονίαι is familiar: it is to be found, for example, in Cleonides *Eisagoge* 19. 4 ff., where rather more detail is given, and where the names of the ἁρμονίαι are again ascribed to οἱ ἀρχαῖοι (19. 7: cf. also 'Bellerman's Anonymous' 62). I shall suggest, however, that Aristides' version has independent interest. What he tells us in the first two passages is found nowhere else.

I shall argue, somewhat indirectly, that Aristides' information has a claim to be considered authentic, and that it helps in certain respects with the interpretation of the text of Aristoxenus himself. I shall approach this conclusion by seeking to establish something about the form in which the information was transmitted to him. It may be too ambitious to make any claims about the actual vehicle of this transmission (though I shall make some tentative suggestions), but it may be possible to reconstruct certain aspects of the manner in which the information was expressed. This will provide some clues to the date of the original source, though that may of course not have been what Aristides had in front of him.

The 'divisions' of 18. 5 ff. form a set of scales displaying marked oddities. Some of them cover an octave, some less, and one a tone more. Their internal structures are sometimes irregular: some also contain gaps where, in the formalized post-Aristoxenian systems, one would expect notes, and some include notes where one would expect an undivided interval. These eccentricities are not (or certainly not all) due to corruption in the text of Aristides, since he himself remarks on several of them; and they are in themselves some evidence of the antiquity of the forms they record. So far as that goes, they might well represent types of ἁρμονία, patterns of tuning, which pre-date the orderly systems of the late fourth century.[3]

[1] The most recent edition is that of Winnington-Ingram (Teubner, Leipzig, 1963). All references to Aristides in this paper are to the pages and lines of that editon.

[2] *Rep.* 398e2–399a4.

[3] The details are discussed in R. P. Winnington-Ingram, *Mode in Ancient Greek Music* (Cambridge, 1936), pp. 21–30. See also the works referred to in his n. 2, 22.

Aristides sets them out in two ways. First, he lists the sequence of intervals which each incorporates. His lists are given in terms of tones, quarter-tones, and so on. This fact suggests one immediate conclusion: his source, whatever it was, did not belong to the 'Pythagorean' tradition which represented intervals as numerical ratios.[4] Two of the intervals mentioned, the diesis or quarter-tone and the *τριημιτόνιον*, three semitones, cannot be given exact equivalents in Pythagorean ratio theory, and no Pythagorean would have agreed that the octave consists of six tones, as this passage requires.[5] Nor would Aristides have had any motive for 'translating' a set of Pythagorean ratios into the terminology of quarter-tones and semitones, giving the nearest equivalents he could find, since he is elsewhere perfectly happy to retain the ratio formulation.[6] Despite the reference to Plato, then, the source is unlikely to have been anyone working in close association with the Academy, whose language and presuppositions are always, in this respect, Pythagorean.[7]

Aristides' second exposition of the divisions is for present purposes unhelpful. It consists of tables of notes displaying the interval structures previously set out in words, and the notes are indicated by the symbols of the so-called 'Alypian' system.[8] Though this system is certainly much older than Aristides, it equally certainly does not go back as far as Aristoxenus, whose main surviving discussion of notation presupposes, as we shall see, a scheme of a totally different type. If the source used the Alypian system, it is probably to be located in the second century B.C. or later. But it seems much more likely that the tables were constructed by Aristides himself, on the basis of information which he had seen in a form more closely corresponding to his first exposition: this appears to be implied in the way he introduces them – *σαφηνείας δὲ ἕνεκεν καὶ διάγραμμα τῶν συστημάτων ὑπογεγράφθω* (19. 1–2). A rudimentary attempt appears to have been made to make all the scales relate, in a more or less orthodox way, to one particular note, that indicated by the symbol C: whoever transferred the interval structures to the tables was evidently anxious to represent at least one note (and there is only one which all the tables share) as 'fixed' in the Aristoxenian sense.[9] But the Aristoxenian analysis is clearly inappropriate to these scales: the representation is the work of someone attempting to interpret disorderly archaisms to an audience schooled in the systematic patterns which theorists had themselves imposed.

Of the two systems of 'ancient notation' preserved at 12. 12 ff. I shall say little at present. The symbols they use are quite different from those of Alypius, and the basis on which they are organized is different too. They apparently represent degrees on a continuum of pitch divided at equal intervals, the first into quarter-tones, the second into semitones. Each covers an octave, and there is no suggestion that this octave had any particular pitch. The notation gives a way of representing divisions of the octave

[4] A careful exposition of the differences between Pythagorean and Aristoxenian approaches to the expression of intervals will be found at Ptolemy, *Harmonics*, 1. 5 and 9 (11–12 and 19–21, During).

[5] See e.g. the Euclidean *Sectio Canonis*, proposition 14.

[6] Very frequently, e.g. lines 4–9, and throughout Bk. III.

[7] Following the example set by *Timaeus* 35b1–36d7. Cf. the qualified approval given to Pythagorean musical theory at *Rep.* 531c1–d4, and Plato's contempt for their 'empiricist' rivals, 531a4–b9.

[8] Named after Alypius (probably third century A.D.), whose *Eisagoge* sets out the tables of this notation in full. But its origin is certainly much earlier.

[9] The 'fixed' notes (*τὰ ἠρεμοῦντα, φθόγγοι ἀκίνητοι, μένοντες*) are those which do not change with changes of genus, and thus provide a constant framework within which variations may occur. The intervals between them are determinate, whereas those involving the 'movable' notes are flexible. See e.g. Aristoxenus, *Harm.* 21. 32 ff., 33. 32–4, 46. 19–23.

at any arbitrary locus. Chailley[10] has argued convincingly that they correspond in all important respects to the system of notation mentioned and criticized by Aristoxenus at *Harm.* 39–40. No trace of their use has been found elsewhere. It seems likely that they reproduce a system used by theorists – almost certainly not by composers – at a date around that of Aristoxenus or a little earlier.

At 15. 8–20 Aristides presents a list of the ἁρμονίαι distinguished by οἱ παλαιοί, which are Mixolydian, Lydian, Phrygian, Dorian, Hypolydian, Hypophrygian, Hypodorian. These are not to be confused with the τόνοι, or transposition scales, of Aristoxenian theory, despite the similarity of their names. They are designated as εἴδη τοῦ διὰ πασῶν, forms or arrangements of the octave – structures much more like 'modes' than keys or transposition scales. That is, they differ from one another, as Aristides' description makes clear, in the order of the intervals which they contain. He distinguishes them as beginning from different notes: but he goes on to say that if in each case you begin from the same σημεῖον, giving it for each ἁρμονία a name corresponding to a note different in respect of function, the succession of notes which follow will make clear the ποιότης of each ἁρμονία.

The point is quite straightforward. The 'functionally named' notes from which the ἁρμονίαι are said to begin are, in order, *hypate hypaton, parhypate hypaton, diatonos* (= *lichanos*) *hypaton, hypate meson, parhypate meson, diatonos* (= *lichanos*) *meson, mese.* These names indicate notes which, in any given genus, stand at fixed intervals from one another. It is legitimate to assume that Aristides is here referring to a scheme applied in the enharmonic genus,[11] in which the intervals separating the notes named are, in order, quarter-tone, quarter-tone, ditone, quarter-tone, quarter-tone, ditone. (In the sequel I shall abbreviate 'quarter-tone' as 'q' and 'ditone' as 'd': 'tone' will later appear as 't' and 'semitone' as 's'.) Since the ἁρμονίαι are explicitly described as octaves, and Aristides gives no hint that their interval structures are in any way unusual, we must suppose that the scales follow the disjunct series, in which the interval above *mese* is a tone, followed by a repetition of the tetrachord q, q, d. It can then be seen that octave scales beginning from the seven notes mentioned will differ in form from one another, following the sequence

Mixolydian	q, q, d; q, q, d; t
Lydian	q, d; q, q, d; t; q
Phrygian	d; q, q, d; t; q, q
Dorian	q, q, d; t; q, q, d
Hypolydian	q, d; t; q, q, d; q
Hypophrygian	d; t; q, q, d; q, q
Hypodorian	t; q, q, d; q, q, d

There is nothing even mildly controversial about this interpretation. It corresponds exactly to what Cleonides tells us in the passage mentioned above. Aristides' remark that the ποιότητες of these ἁρμονίαι will be revealed if we begin them all from the same σημεῖον also seems readily comprehensible: he means that if we begin them all at the same point of pitch, but designate this differently 'in respect of function' (i.e. in respect of its location in the series of named notes which mark out the sequence of intervals), the natural sequence of functionally named notes will map on to one and the same range of pitch, in such a way as to make their different structures easy to compare.

[10] J. Chailley, 'La notation archaïque grecque d'après Aristide Quintilien', *Revue des Etudes grecques* 86 (1973), 17–34. More briefly in his *La Musique Grecque Antique* (Paris, 1979), 121–5: see especially 123, n. 2.

[11] Aristoxenus 2. 7–25. Aristides is not explicit on this point.

Now it is obvious that this set of structures belongs to a later date and a higher level of systematization than do the divisions allegedly relating to the ἁρμονίαι of the *Republic*. But equally our evidence indicates that they pre-date Aristoxenus. Though the orderings of intervals which they embody in no way conflict with his doctrines, their attribution to οἱ ἀρχαῖοι by Cleonides, who follows Aristoxenus slavishly, makes it clear that they were not Aristoxenus' invention. Nor does anything in Aristoxenus' extant work suggest that he was much concerned with such constructions: the focus of his analysis lay elsewhere, as I shall explain below. Further, there are references in Aristoxenus himself to structures discussed and explicated by his predecessors, which can hardly be any but these. He tells us that the 'harmonicists', by which he means all his precursors, failed to study the nature and interrelations of magnitudes other than the octave, confining themselves to ὀκταχόρδων ἐναρμονίων (2. 17). In another passage (36. 30) he speaks of them as considering nothing but αὐτῶν μόνον τῶν ἑπταχόρδων ἃ ἐκάλουν ἁρμονίας: the editors have unanimously and probably rightly emended ἑπταχόρδων to ἑπτὰ ὀκταχόρδων.[12] They are not relying only on 2. 17, and on the general consensus that the word 'ἁρμονία' almost always designates the octave in works of this period.[13] A more important clue comes from 6. 13 ff., where Aristoxenus explains that one of his predecessors, Eratocles, 'attempted to enumerate the arrangements of the (enharmonic) octave, displaying them by moving the intervals around cyclically' (τῇ περιφορᾷ τῶν διαστημάτων, 6. 24). An 'arrangement' (σχῆμα, elsewhere also εἶδος) in Aristoxenus' terminology is an ordering of a collection of intervals whose number and magnitude are already determined (74. 9–17). The reference to 'cyclic' rearrangement may reliably be interpreted as follows. If we begin from the standard structure of the enharmonic octave, q, q, d; t; q, q, d, we can generate new arrangements by 'moving the intervals around', that is, by first moving the lowest interval from the bottom of the series to the top, producing q, d; t; q, q, d; q, then the second interval, giving d; t; q, q, d; q, q, and so on. As Aristoxenus says, this will generate seven arrangements (6. 30); and he asserts that although it is only this group of seven which Eratocles 'enumerates' (ἐξαριθμῆσαι, 6. 22) and 'displays' (δεικνύς, 6. 25), he has done nothing, though he should have done, to rule out the legitimacy of arrangements not generated by this procedure. (Examples would be q, q, t, d, q, q, d, or q, q, d, t, d, q, q, and there are many others.)

It is obvious that this procedure, and the octave ἁρμονίαι which it generates, correspond exactly to what Aristides is talking about at 15. 8 ff. Though he puts it differently, the procedure which generates his seven ἁρμονίαι is precisely that of cyclic rearrangement; and there is no serious room for doubt that the two authors are discussing the same system. The question I want to focus on is whether the additional manoeuvre mentioned by Aristides, that of starting all the ἁρμονίαι from the same σημεῖον for the purposes of comparing their structures, also finds an echo in Aristoxenus. I believe that it does, and that this echo has interesting implications for the question about transmission with which we began.

The issue will hang on our understanding of a procedure which Aristoxenus ascribes to his predecessors, and which he calls *katapyknosis*. The term means, roughly, 'close

[12] See the edition of R. da Rios (Rome, 1954), app. crit. to 46. 9, and her translation, p. 53, n. 2. Chailley's proposal (op. cit. p. 38) to retain the MSS reading runs into difficulties which I shall discuss later.

[13] Asserted by e.g. Nicomachus, *Enchiridion* 9; cf. Aristides Quintilianus 15. 8–10, Plato, *Rep.* 617b6–7. But there is some vacillation about this: cf. Aristotle, *Metaph.* 1093a14. The supposedly older heptachord scales were at least sometimes called ἁρμονίαι, e.g. in ps.-Ar. *Problems*, 19. 7, 918a13, and 25, 919b21. The divisions of Aristides Quintilianus 18–20 constitute another kind of exception (see 18. 6, 19. 8).

packing' or 'compression'. The references in Aristoxenus make it clear that it had an association with certain *diagrammata* on which earlier theorists had represented the interval structures of the enharmonic octave (7. 22 ff., 28. 1). Whether the term refers to the 'close packing' of intervals in the diagram or the 'compression' of the diagram itself is a question which will concern us later.

There is general agreement about the approximate nature and function of the procedure. The diagrams apparently took the form of a line divided into equal segments, each representing a quarter-tone. The aim was to map on to the diagram the structures of accepted (or postulated) enharmonic forms of the octave. Whichever interpretation of the word '*katapyknosis*' we choose, it evidently designates the attempt to fit all the relevant intervals and arrangements of the scale into as small a compass as possible, presumably with a view to simplifying the task of quantifying and comparing – perhaps visually – the interval structures of each.

So much is relatively uncontroversial. Problems arise when we try to reconstruct in detail the way in which the mapping was done. The reconstruction offered by Macran in his edition of Aristoxenus[14] looks plausible, but meets with difficulties; I have not come across any equally detailed alternative.

Any reconstruction must try to take into account the following points.

(i) The diagrams were used only to represent enharmonic octave *systemata*, though they actually displayed (ἐδήλου: the sense seems to be 'could have accommodated') 'the whole of melodic order' (*τὴν πᾶσαν τῆς μελῳδίας τάξιν*). In the context this last phrase must mean 'comparable *systemata* in the other genera as well as the enharmonic' (2. 12 ff.).

(ii) They do not give any real help with problems concerning modulation (*μεταβολή*), nor were they designed to do so; but 'certain of the harmonicists have briefly touched on this part of the science quite accidentally (*κατὰ τύχην*), while not actually addressing themselves to this matter, but seeking to compress (or "closely pack") the diagram (*καταπυκνῶσαι βουλομένοις τὸ διαγράμμα*)'. (See 7. 22 ff.: cf. 38. 3.)

(iii) The diagram is said to involve a series of 28 consecutive dieses, quarter-tones. This, Aristoxenus asserts, represented continuity, *τὸ ἑξῆς*, to the harmonicists (27. 34–28. 10). Macran appears to think that this means that the 28-diesis range was full; that is, every pitch on it was occupied by at least one note in one form of the scale or another. I shall offer an alternative interpretation. Attempts have been made to emend '28' to '24', on the grounds that there are only 24 dieses in an octave.[15] Here Macran's defence of '28' is probably on the right lines, but the question may be left open for the moment.

Little more can be gathered from Aristoxenus' direct references to *katapyknosis* and the diagrams, but it is worth recalling two other claims which he makes about his predecessors, which we have already noticed. They are

(iv) that Eratocles attempted to enumerate the arrangements of the octave, 'displaying them by moving the intervals around cyclically' (6. 13 ff.), and

(v) that none of his predecessors made any systematic study of 'magnitudes', *μεγέθη*, other than the octave. I shall also need to refer to the claim that

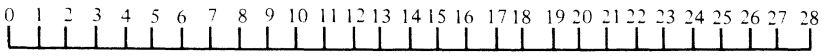
(vi) the school of Eratocles noted the fact that the melodic sequence 'splits in two as it moves in either direction from the interval of a fourth' (5. 9 ff.). This is usually, and I think rightly, understood as a reference to the alternatives of conjunction and disjunction as methods of linking successive tetrachords. When we reach the highest

[14] *The Harmonics of Aristoxenus* ed. H. S. Macran (Oxford, 1902), pp. 229–32. What I shall call his 'reconstruction' is constituted by the tables of notes on 231.

[15] By Ruelle, following Meibom's note. See da Rios op. cit., app. crit. to 36. 5–6.

note of the first tetrachord we may continue the scale *either* by treating the note we have reached as the lowest note of the second tetrachord (conjunction), *or* by beginning the second tetrachord on a note at an interval of a tone from the last note of the first (disjunction) (see 58. 14–59. 5). An enharmonic tetrachord typically has the form q, q, d: a sequence of such tetrachords may have the conjunct form q, q, d; q, q, d;..., or the disjunct form q, q, d; t; q, q, d;...Aristoxenus criticizes the Eratocleans' work on this subject on various grounds; these, though of interest in their own right, need not concern us here.[16]

In the light of these points we should first consider Macran's reconstruction. He sets out his version of the seven octachords, labelled with the names found in Aristides and Cleonides, on a set of musical staves. It will be more convenient for present purposes to represent them differently, and perhaps more in the manner of the harmonicists themselves. Their diagram was a line, divided equally into quarter-tones. If we draw such a line, with the bottom of the scale to the left, we can number the notes marked on it, with 0 as the first, and refer to the notes in our scales by their numbers on the diagram.



For each of the *systemata* (or *harmoniai*) I shall give first its interval structure, which corresponds exactly with that implied by Aristides and set out previously, and secondly the numbers corresponding to its notes according to Macran's version.

Mixolydian	q, q, d; q, q, d; t	4, 5, 6, 14, 15, 16, 24, 28
Lydian	q, d; q, q, d; t; q	2, 3, 11, 12, 13, 21, 25, 26
Phrygian	d; q, q, d; t; q, q	0, 8, 9, 10, 18, 22, 23, 24
Dorian	q, q, d; t; q, q, d	5, 6, 7, 15, 19, 20, 21, 29
Hypolydian	q, d; t; q, q, d; q	3, 4, 12, 16, 17, 18, 26, 27
Hypophrygian	d; t; q, q, d; q, q	1, 9, 13, 14, 15, 23, 24, 25
Hypodorian	t; q, q, d; q, q, d	6, 10, 11, 12, 20, 21, 22, 30

The scheme works well at several points. It plainly conforms to the requirements of points (i) and (v), and also to those of (iv), since the different *systemata* are generated by cyclic rearrangement. Macran also takes it to reflect the claims made in (ii), which he takes to imply the thesis that the arrangement of scales in the *katapyknosis* diagrams was inconsistent with the possibility of modulating from one to another, according to the canons which Aristoxenus laid down. Unfortunately we know little directly about Aristoxenus' views on modulation. Macran, however, is arguably right in supposing that the intervallic relations between the fixed notes of one *systema* and another were important, and particularly those between what he rather misleadingly calls 'tonics', the *mesai* in the sense distinguished by Ptolemy as *mesai kata dunamin*.[17] In these *systemata* the *mese* is in each case the note immediately below the interval of a tone. If we may, as Macran does,[18] read back the views of Bacchius on modulation

[16] He argues that they have not discussed the question whether bifurcation of the series may take place after just any form of the (enharmonic) fourth (e.g. q, d, q) or only after one privileged form (this will be q, q, d). Various propositions in Bk. III seem expressly designed to fill this gap, and to prove the impropriety of disjunction after any other arrangement of the fourth (see 65. 30–66. 8, 66. 9–17, 71. 23–72. 11). But Aristoxenus does not say, and almost certainly does not mean, that the Eratocleans posited disjunction at any points which he himself does not recognize: he means that they have omitted to show *why* such rogue disjunctions are impermissible.

[17] Ptolemy, *Harmonics* 2. 5 (51, During). On the importance of relations between *mesai* cf. Cleonides, *Eisagoge* 23. 14 ff.

[18] Macran, op. cit. pp. 262–6. See also Cleonides 26. 19 ff. and Aristides Quintilianus 22. 11–26.

into Aristoxenus, we reach (after some complex working) the conclusion that the only *systemata* between which modulation, direct or indirect, is possible, are those whose *mesai* are separated by a semitone or a multiple of a semitone. In Macran's reconstruction the *mesai* of the octachords are, in order, 24, 21, 18, 15, 12, 9, 6. The interval separating each from its neighbour is thus three-quarters of a tone, and hence modulation between adjacent octachords is impossible. In fact it turns out to be impossible between any pair of octachords, since none of the intervals at which *direct* modulation is permitted occurs. (These are the tone, the fourth, the fifth, and five tones,[19] none of which is a multiple of three-quarters of a tone.)

So far so good: but there are two major difficulties. One is that the reconstruction patently conflicts with point (iii). Certainly it does involve 28 consecutive dieses, but only in the sense that it involves at least that number, since it incorporates 30. If the diagrams Aristoxenus met ran to 30 dieses, he would undoubtedly have said so, since part of his polemical point is that they string together far too many. Macran, for reasons he does not explain, focuses only on one octave within the total range of his structure, running in his formulation from F to f, and in the present one from 4 to 28. This contains 24 dieses, but we are not told why we should leave out of account the notes numbered 0, 1, 2, 3, 29, and 30. His defence of the reading '28' at 28. 7, which I shall look at further below, implies that a set of octave scales would have been represented within a range of 24 dieses. (The additional four are explained by the hypothesis that one at least of the scales represented covered the range of an octave plus a tone.) Why then does his reconstruction demand 30?

Secondly, Macran gives no explanation of why he arranges the seven systems at these particular distances from one another. Nothing in the information we have underwrites the spacing of *mesai* at intervals of three-quarters of a tone; and although the series of initial notes, 4, 2, 0, 5, 3, 1, 6 displays a certain pattern, it does not look like a pattern that would have been deliberately sought for its own sake. It is hard to resist the conclusion that he has spaced them in this way precisely in order to generate the result that modulation is impossible. But it can hardly have been the intention of the harmonicists to rule out modulation, and I can conceive of no other rationale for choosing this particular set of relations. It seems most unlikely to have been adopted merely by chance. Further, Aristoxenus does not say that their diagrams make modulation impossible: he says that they said something about it briefly and accidentally, while actually pursuing a different objective. Aristoxenus pulls no punches; and if the diagrams genuinely had the consequence which Macran's version does, he would hardly have been able to resist drawing attention to the fact.

Any rival reconstruction must begin by trying to reduce the number of consecutive dieses. This cannot be done without a fairly radical revision of Macran's approach to the problem. The *systemata* must each cover an octave, 24 dieses. There are seven of them, so that if we are to place their starting points on different notes, a minimum of a diesis apart, we are bound to add six dieses to the total range covered, producing Macran's 30.

One solution would be somehow to abbreviate the *systemata*. The obvious way of doing so is to take them to be not the octave-spanning octachords of the disjunct system, such as q, q, d; t; q, q, d, but heptachords of the conjunct system, q, q, d; q, q, d, which span a tone less. Plainly this will reduce the range of the diagram, though it will make it 26 dieses, not 24 or 28. But the suggestion will not do. Aristoxenus' references to octave scales are clear (even if the text as we have it speaks in one place of heptachords rather than octachords); more decisively, cyclic rearrangement of the

[19] Macran, op. cit. p. 266.

intervals of the conjunct series will not produce seven arrangements, but only three, q, q, d; q, q, d and q, d; q, q, d; q and d; q, q, d; q, q. With the next move we are back to the original form.

A second manoeuvre would be to eliminate one of the two bounding notes of each of the octachord octave sequences, on the grounds that since they stand at an octave to one another they are effectively the same note, and one must be redundant. The higher of the two is to be considered as the starting point for the second octave, not the end of the first. This approach has two advantages. It would give an explanation for the apparent vacillation between talk of octachords and talk of heptachords. Secondly, it enables us to reduce the span of the diagram to 28 dieses. Macran's own reconstruction cannot be refurbished in this way, since if we simply remove the higher octave note from each of his scales we shall again be left with a range of 26 dieses, not 24 or 28. There are, however, a number of ways in which it can be done, of which the most attractive is that which retains Macran's and Aristides' order of ἁρμονίαι but spaces their starting points at intervals of one diesis, the highest belonging to Mixolydian. This will yield the sequence

Mixolydian	6, 7, 8, 16, 17, 18, 26 (30)
Lydian	5, 6, 14, 15, 16, 24, 28 (29)
Phrygian	4, 12, 13, 14, 22, 26, 27 (28)
Dorian	3, 4, 5, 13, 17, 18, 19 (27)
Hypolydian	2, 3, 11, 15, 16, 17, 25 (26)
Hypophrygian	1, 9, 13, 14, 15, 23, 24 (25)
Hypodorian	0, 4, 5, 6, 14, 15, 16 (24).

The numbers in parentheses complete the octaves, and are deemed redundant on the basis discussed above.

We may tighten up the diagram more thoroughly. According to Macran's conception of τὸ ἑξῆς, continuity, the scales deployed should make use of 24 or 28 consecutive dieses. But in our set of *systemata* three positions in the series are empty, numbers 10, 20 and 21. My calculations have yielded no sequence, built on the present principle and organized according to a non-arbitrary system of spacing and ordering, which displays no such gaps. But in certain sequences, including this one, they can be filled if we add in to the diagrams the notes of the conjunct series.[20] Such an addition would be consistent, as we have seen, with the work of at least one group of Aristoxenus' predecessors. Thus we have seven abbreviated octave scales, each with its five-tone conjunct variant, represented on a continuous series of 28 dieses; since they begin on different notes and have their *mesai* quite intelligibly placed, they might be conceived as having something to do with modulation; and the series of consecutive dieses separating their initial notes indicates a perfectly sensible methodology behind their arrangement – in this they differ markedly from Macran's.

But I do not think that the hypothesis from which they begin can stand. Our authorities are almost unanimous in treating octave scales as ones which include the highest note, rather than merely implying it, and as containing eight notes, not seven.[21]

[20] The notes of the conjunct versions of these *harmoniai* will be as follows: Mixolydian, 6, 7, 8, 16, 17, 18, 26; Lydian, 5, 6, 14, 15, 16, 24, 25; Phrygian, 4, 12, 13, 14, 22, 23, 24; Dorian, 3, 4, 5, 13, 14, 15, 23; Hypolydian, 2, 3, 11, 12, 13, 21, 22; Hypophrygian, 1, 9, 10, 11, 19, 20, 21; Hypodorian, 0, 1, 2, 10, 11, 12, 20. Thus 10 and 20 are supplied by Hypophrygian and Hypodorian, 21 by Hypolydian and Hypophrygian.

[21] The only seven-note scales generally referred to are those of the conjunct system, spanning five tones. The alleged innovator who 'added the eighth note' is commonly thought of as thereby completing the octave. See e.g. Nicomachus, *Enchiridion* 5. The main exception is ps.-Ar.

The trick of abbreviation by which our result was achieved is nowhere mentioned, and had it been in use we would surely have heard about it, since it is such a convenient and economical device.[22]

I suggest that we can properly resolve the difficulties only be re-examining our assumptions at two crucial points. The first concerns the relevance of the harmonicists' procedure to questions about modulation. I have already noted that Macran's interpretation of Aristoxenus' comments is unjustified: the arrangement need not be such as to make modulation impossible. It should now be added that it need not even be such as to require that the various ἁρμονίαι begin from different notes. Quite the contrary: one way of approaching the topic of modulation between different octave forms would be to start each form on the same note, thereby displaying facts of three kinds: (*a*) that if a melody begins in e.g. the Dorian ἁρμονία, and moves from the position mapped as the lowest note through a quarter-tone upwards, no μεταβολή is involved, whereas if it moves, for instance, through a ditone, it has shifted to Phrygian or Hypophrygian; (*b*) that – beginning again from some particular ἁρμονία – a given interval placed at one point in the system will not generate modulation, whereas placed at another it will;[23] and (*c*) that there are only certain positions at which μεταβολή between a given pair of *systemata* is consistent with the pattern of relations which the diagram displays. Since, for example, the note one diesis from the bottom does not occur in Phrygian, we cannot modulate into the Phrygian from the Dorian at that point in the series.

This approach to modulation is certainly not Aristoxenian. As far as we can judge, the question he asks is not 'How can we move from one form of the octave to another?', but 'At which intervals can we move from a *systema* in a given range of pitch to a *systema* of identical form at another range of pitch?'[24] The relationally pitched set of structurally identical *systemata* are the τόνοι, or 'keys',[25] and it is these, not the ἁρμονίαι, which are central to Aristoxenus' theory of μεταβολή. But the questions are in a certain sense equivalent. In practice, moving from the Dorian ἁρμονία based on a certain pitch to the Phrygian ἁρμονία based on the same pitch is equivalent to remaining in the Dorian arrangement of the octave and moving its starting point upwards through two tones. Thus a diagram which began all its ἁρμονίαι at the same pitch would satisfy the condition of saying something 'by chance' about modulation, and also fail to approach it in the way which Aristoxenus believes to be practically illuminating. His objective is to relate τόνοι and modulations to 'regions of the voice':[26] that is, I think, to show how the centre of gravity of a musical passage may be shifted upwards or downwards in pitch through the device of modulation.[27] That of the harmonicists is merely to pack the ἁρμονίαι, for purposes

Problems 19. 32, 920a 14–18; cf. 7, 918a 13–18 and 47, 922b 3–9, which display some uncertainty on the matter. But the ἁρμονίαι to which these passages refer are evidently thought of as much older than any which are our present concern.

[22] The closest analogy is provided by Ptolemy's limitation of the τόνοι to seven, ruling out repetition at the octave (2. 8). But his argument is evidently a new one: in the thirteen Aristoxenian τόνοι the last is at an octave from the first (Cleonides 25. 4–26. 15).

[23] cf. Aristoxenus 34. 8–11.

[24] Aristoxenus 7. 10–8. 3.

[25] Cleonides loc. cit. Cf. Aristoxenus 7. 23, 37. 8–38. 5.

[26] Aristoxenus 7. 10–8. 3.

[27] The importance of this for the character of the melody is emphasized at 7. 13–16. It is arguable that in this respect Aristoxenus is claiming for his τόνοι some of the ethical significance earlier assigned, as Aristides Quintilianus (15. 19–20) and many others tell us, to the old ἁρμονίαι. He seems, however, to have avoided the wilder excesses of certain versions of this theory (cf. 31. 16–29).

of structural comparison, into as close a space as possible. The fact that they end up with something which relates to questions about modulation is merely an unintended by-product of their procedure; and this is precisely what Aristoxenus tells us at 7. 10–8. 3.

Secondly, I do not think it right to insist that the series of 24 or 28 dieses be unbroken, in Macran's sense, and that a series displaying gaps where no note falls is thereby disqualified. The main passage underlying Macran's assumption is worth translating in full. 'Continuity (*τὸ συνεχές*) is not to be sought in the way in which the harmonicists try to set it out in the *katapyknoseis* of the diagrams, displaying as successive with one another those notes which happen to be separated by the smallest interval. So far is the voice from being able to sing 28 dieses consecutively that it cannot even add a third diesis, but in its progress upwards sings at least the remainder of the fourth – anything less is impossible' (27. 34–28. 13). It is quite clear that here, as in some other cases, Aristoxenus is attacking his opponents on inappropriate grounds.[28] That is, he criticizes them for describing as *ἑξῆς ἀλλήλων* a series of notes each separated from the next by one diesis, on the grounds that no one could possibly sing such a series. For Aristoxenus, two notes are directly successive with one another if no legitimate note can in musical practice be placed between them. That is why he asserts that after an upwards series of two dieses the voice must sing no less than the remainder of the fourth, a ditone, since, so he argues,[29] there is no legitimate form of musical scale in which the interval immediately above the pair of dieses is less than that. But the tying of the notion of 'continuity' to that of a legitimate musical scale derives from a very special understanding of continuity itself, and one which there is no reason to believe that the harmonicists shared. It is wildly improbable that they thought of their 28-diesis series in this manner. They would have done so only if they thought of that series as a scale reflecting patterns of intervals employed in musical practice: and their scales, in this sense, are the seven *ἁρμονίαι*, none of which demands any such vocal gymnastics. The sequence of dieses merely represents *theoretical* continuity, in the sense of that series of notes between which no usable note could fall no matter what form of scale is considered legitimate. According to this conception, there simply are no notes between the points marked, and it is in this sense that the series represents *τὸ ἑξῆς* or *τὸ συνεχές*.[30] Aristoxenus' criticism would have been better expressed by saying that their conception of continuity cannot be used for the purposes for which he wishes to use his own; and that is perfectly true.

'Continuity' is then simply the metric, the theoretically plotted sequence of minimally spaced positions on to which the actual scales are to be mapped. There is nothing in this to suggest that we should be worried by the presence, in any postulated reconstruction, of positions in the continuous sequence on which no note of an actual scale falls. It also enables us to understand the expressions *ἐν ταῖς τῶν διαγραμμάτων καταπυκνώσεσιν* (28. 1) and *καταπυκνῶσαι τὸ διάγραμμα* (7. 31–2) in what seems to me their natural sense, referring not to a process of 'filling up the diagram' or 'closely packing' the intervals it displays, but 'compressing the diagram' into the smallest possible range of pitch.

This brings us to one further point. If, as Aristoxenus tells us, their objective was

[28] I have argued this in '*οἱ καλούμενοι ἁρμονικοί*: the predecessors of Aristoxenus', *Proceedings of the Cambridge Philological Society* n.s. 24 (1978), 1–21, especially 15–17.

[29] 67. 10–25.

[30] Plato, *Rep.* 531 a 4–b 1 comments on the pursuit of the least interval. Cf. Aristotle, *Metaph.* 1016 b 18, 1052 b 20, 1083 b 33. Contrast Aristoxenus' implication that theoretically (not melodically) speaking there can be no least interval, 15. 7–12.

to 'compress the diagram', and if what they were dealing with were octave scales, there could have been no sensible reason for choosing a sequence of 30 dieses or 28 or 26. The only rational choice is 24, the number of dieses in the octave. I do not mean to imply that '28' at 28. 7 should be altered: as the *difficilior lectio* it should certainly be retained if possible. But let us begin with the 24-diesis diagram.

An interesting feature of Aristoxenus' forms of expression at 7. 31 and 28. 1 is their implication that the harmonicists began from something which was not the compressed diagram, but which could be turned into it. 'Wanting to compress the diagram' and 'in the compressions of their diagrams' are phrases which suggest the existence of an

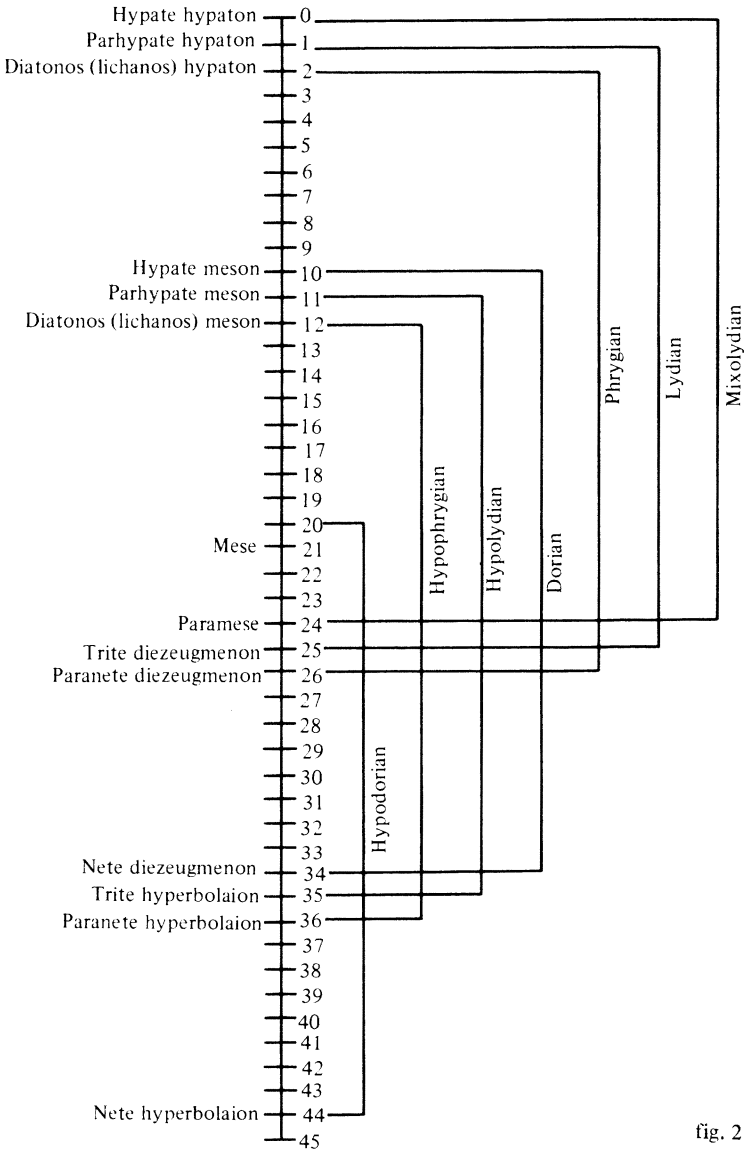
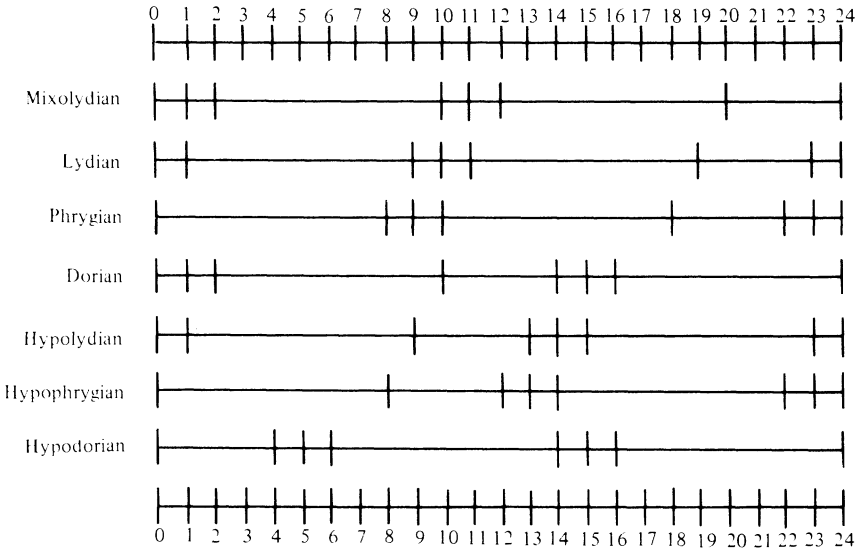


fig. 2

uncompressed diagram upon which the process of compression was then performed. Let us suppose, then, that the treatises which Aristoxenus saw contained at least two kinds of diagram, the first uncontracted, the second compressed. If we now return to Aristides, we find that his account of what I take to be the same matter has exactly the same feature. He begins from the notion that each ἁρμονία starts from a different note – Mixolydian from *hypate hypaton*, Lydian from *parhypate hypaton*, and so on. If we represent this in diagrammatic form, mapped on to a series of consecutive dieses, we shall produce something which might look like fig. 2.

Aristides then goes on to say that the ποιότητες of the ἁρμονίαι will be revealed if we begin them all from the same σημεῖον. If we take a σημεῖον here to mean an arbitrary pitch-point designated merely as the first in a sequence, and again map the ἁρμονίαι on to a series of consecutive dieses, we shall get the following result (fig. 3):



It seems to me likely that this is at least very close to the compressed diagram to which Aristoxenus refers. But we have arrived at it by a fairly complex argument, and I do not suppose that Aristides reached his account of it by the same means, that is, by derivation from hints in the text of Aristoxenus as we now have it. He is describing something he has seen, or of which he has had a fairly direct report. It is obviously relevant that he, alone among our authorities, records an allegedly ancient quantitative system of notation which did precisely the job which the diagram demands, that of representing by a symbol each note in a quarter-tone series. I suggest that when he talks of beginning each ἁρμονία from the same σημεῖον, it is of the symbols in this system of notation that he is thinking.

The fact that Aristides' description fits so well with the procedure hypothetically reconstructed from Aristoxenus, and the fact that he had access to an ancient and appropriate system of notation, may give us more confidence in the value of a coincidence between one of the ἁρμονίαι he associates with the *Republic* and an oddity already noticed in the text of Aristoxenus. (Such confidence will of course only be justified if the source for all three of his passages is the same.) Aristoxenus, as we have seen, talks about a continuum of 28 dieses. One of Aristides' ancient divisions, that

of the Dorian, exceeds the octave by a tone, and would thus require a 28-diesis framework. This fact is alluded to by Macran,[31] though he fails to mention the authority on which he relies. He uses it to defend the reading '28', thereby implying that octave scales would be represented, as above, on a diagram spanning 24 dieses, and implicitly undermining his own reconstruction. But the defence itself is perfectly in order, just so long as we may reasonably suppose that the source for the divisions is the same as that for the much more highly systematized ἁρμονίαι of 15. 8 ff. and the notation presupposed by the *katapyknosis* diagrams.

Two considerations give this supposition a little support. One is the fact mentioned previously, that the source for the divisions cannot be Pythagorean. The terms in which they are presented are consistent with those used both by Aristoxenus himself and his successors and by the authors of the *katapyknosis* diagrams. The second is Aristoxenus' insistence on the 'empiricist' stance of his harmonicist predecessors. He accuses them on several occasions of attempting to 'enumerate' various phenomena, through the use of perception, and of failing to demonstrate the truth of what they assert – of failing, that is, to derive truths about μέλος from acceptable first principles. Further, he mounts a sustained and vitriolic attack on those who think that the object of the science of ἁρμονική lies in notation, in the writing down (in a quantitative form, as his criticisms make clear) of melodies and scales.[32] Taken together, these remarks imply that at least one of the activities of the harmonicists was the attempt to develop a precise notation suitable for the quantitative expression of the forms of *systema* actually used in current musical practice. While the ἁρμονίαι of Aristides 15. 8 ff. bear obvious marks of artificial schematization, this is not true, or certainly less true, of the 'ancient divisions'. It is a reasonable inference that the schematizations began from an 'empirical' survey of the data of current musical practice, and sought to tidy it up into a systematic shape. Such a survey may be presumed to have involved writing down the data of practice in a form which made comparisons possible between them and the subsequent theoretical constructions.

When Aristides introduces the 'ancient divisions', he does not say that they *are* the old ἁρμονίαι: he says that they are the divisions αἷς καὶ οἱ πάνυ παλαιότατοι πρὸς τὰς ἁρμονίας κέχρηνται (18. 5–6). I take this to mean 'which persons belonging to extreme antiquity used to represent the *harmoniai*', and it suggests at best a non-committal attitude to the question whether they represented them accurately. Aristoxenus, in his references to his predecessors, accuses them among other things of failing even in their perception of the phenomena. One such passage is 6. 14 ff., where, speaking of Eratocles, he says

ὅτι δ' οὐδεὶ εἴρηκεν ἀλλὰ πάντα ψευδῆ καὶ τῶν φαινομένων τῇ αἰσθήσει διημάρτηκε, τεθεώρηται μὲν ἔμπροσθεν ὅτ' αὐτὴν καθ' αὑτὴν ἐξητάζομεν τὴν πραγματείαν ταύτην.

The backwards reference given here is not to anything in the *Harmonics* as we have it. The likelihood is that it is the same as the reference of 2. 26–30,

ὅτι δ' οὐδένα πεπραγμάτευνται τρόπον οὐδὲ περὶ αὐτῶν τούτων, ὧν ἡμμένοι τυγχάνουσι, σχεδὸν μὲν ἡμῖν γεγένηται φανερὸν ἐν τοῖς ἔμπροσθεν ὅτε ἐπεσκοποῦμεν τὰς τῶν ἁρμονικῶν δόξας.

This clearly designates a separate work, which we no longer have. Not only must it have included a fairly full account and discussion of *katapyknosis*; 6. 14 ff. makes legitimate sense only if he also argued there that certain of their representations of

[31] Macran, op. cit. pp. 252–3.

[32] On their failure to give demonstrations see especially 32. 27–31. For his criticism of the thesis that the aim of harmonics is notation see 39. 4–41. 24.

the phenomena, the actual data of practice, were mistaken. This carries the strong suggestion that it recorded the 'divisions which they employed' in representing those of the *systemata* in actual use which they attempted to notate.

The excellence of fit between Aristides' account of the seven octachords and Aristoxenus' remarks about *katapyknosis* is certainly too close to be coincidental. Either Aristides was equipped with a reliable independent account of the same activities which Aristoxenus mentions, or he was relying on a fuller description given by Aristoxenus himself. Aristides' authority is strengthened by his having recorded the relevant system of notation: it is worth remarking that the nature and existence of this system is nowhere used by Aristides for argumentative purposes – he merely records it and passes on, and has apparently no motive for setting it down beyond the mere fact that he had a copy in front of him. The Dorian *harmonia* of 18. 13–15 gives us the only plausible clue we have to why Aristoxenus apparently wrote '28' and not '24' dieses at 28. 7. If the clue points in the right direction, it indicates that Aristoxenus had seen, and taken note of, a diagram representing a scale which displayed this eccentric additional tone. Apart from the internal details of the ancient divisions and of the system of notation, there is nothing in Aristides at which Aristoxenus does not hint. The detail, however, is such that Aristides cannot have derived it directly from anything in the works of Aristoxenus which we still possess: at several points, as we have seen, it serves to answer questions which the text of Aristoxenus poses, and to explain the reference of Aristoxenus' obscure and inexplicit allusions.

It remains possible that Aristides' source is independent of Aristoxenus. It has been suggested, by Isobel Henderson for example,[33] that the source for the ancient divisions was an early commentary on the *Republic*. I have tried to give reasons, however, for thinking that the source for all three passages is the same, and it is unlikely that any such commentary would have included the information given at 12. 12 ff. and 15. 8 ff. One might also have expected such a commentary to express itself in Plato's own terminology of 'Pythagorean' ratios, not in that of the 'empiricists' whose work he dismissed with derision at *Rep.* 531a4–b9. If there was a single source for all three passages, it might conceivably have been a copy, or an epitome, of a work by one of the harmonicists themselves. In that case, however, one would have expected a greater difference of emphasis between Aristides' account and Aristoxenus'. It would be merely a coincidence that both accounts cover the same ground in greater or less detail, and that Aristides gives details at precisely the points where Aristoxenus' remarks become vague, and hung about with allusions to an earlier work of his own. All the salient features of Aristides' account are explained if we suppose that his source was a copy, or some later reduction, of Aristoxenus' own pamphlet *On the Opinions of the Harmonicists*. If this is correct, we can repose some confidence in Aristides' record of the notation of the pre-Aristoxenian theorists, and in the reconstruction of *katapyknosis* which I have suggested; and while my conclusion would imply that the 'ancient divisions' are indeed more or less contemporary representations of *harmoniai* used in Plato's day or earlier, we may retain a degree of scepticism about the accuracy with which they represented them. It is a scepticism which Aristoxenus appears to have shared.[34]

University of Warwick ANDREW BARKER

[33] *The New Oxford History of Music*, vol. 1 ed. E. Wellesz (London, 1957), 349 n. 4.

[34] This paper has not attempted to give a general account of the ancient *harmoniai* and their development. The subject is difficult and controversial. For a select bibliography see S. Michaelides, *The Music of Ancient Greece, An Encyclopaedia* (London, 1978), pp. 128–9.